Friedmann-Lemaître equations and periodic Cosmos

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"Nothing is born or perishes, but already existing things combine and then separate again."
in Fragments - Anaxagoras (Vth century BC)

***Abstract*:**The initial aim of this essay was to try understanding the reasons to the apparent discrepancy between the theoretical and "observed" behaviors of the Hubble parameter that appeared recently: always decreasing in the first case, decreasing then increasing in the second. This research finally brought us to build a cosmological model of periodic cosmos reconnecting with the ancient wisdom of the quotation given at the beginning of this essay.

***Keywords*:** Friedmann equations; Hubble parameter; Standard Model; Big Bang; periodic cosmological model

Introduction

The last determination of the Hubble parameter shows a discrepancy between the variations of the exact solution to the Friedmann-Lemaître equations and the observed variations: after being decreasing for a long time, the Hubble parameter is now increasing (cf [2] [1]) and that is not compatible with the variations of the exact solution which is decreasing for all time τ .

The Friedmann-Lemaître equations are written in cosmic time τ, proper time of the universe line $\mathcal{T}$ of any Cosmic Observer, [3], but, on the other hand, the experimental determinations of the Hubble parameter are done relatively to "earth time", denoted apparent t in this essay. This identification of these two times, not specified but nevertheless effective in the first section, actually supposes that $\frac{d\tau}{dt} = 1$ for all t, i.e. that $\mathcal{T}$ and the one dimensional Euclidean space $\mathbb{R}$ be isometric and not only smooth homeomorph . We will envisage here that it is the cause of discrepancy and that the identification is only and approximatively possible for "small" times.

Guiven the quotation given at the beginning of this essay to which we subscribe and the incontestability of the Standard model, this path will brings us to postulate that the cosmic time curve is a periodic curve and then to define it and do a first study of its consequences and their compatibility with experimental observations and Standard Model.

Preamble

We will systematically consider here a space-time
of zero spatial curvature
which admits a singularity in its past (Big Bang)
dominated by a constituent governed by the state equation $p = \omega c^2 \rho$

Notations are those of [3] and all this essay is based on thr following postulate:

1^st^ Postulate:*Any singularity of the space-time is the expression of a mathematical singularity of the solutions to the Friedmann-Lemaître equations and vice versa.*

1.Solutions to the Friedmann-Lemaître equations

This essay being totally based on the Hubble parameter, we need first determining its intrinsic expression.

In the framework specified in preamble, the Friedmann-Lemaître equations are written ([3]: p 198):

$$\left|\begin{array}{l}\left(\frac{\dot{a}}{a}\right)^2=\frac{8\pi G}{3}\rho+\frac{c^2\Lambda}{3}\\ \frac{\ddot{a}}{a}=-\frac{4\pi G}{3}\left(1+3\omega\right)\rho+\frac{c^2\Lambda}{3}\end{array}\right.$$

Expressed relatively to the Hubble parameter $H=\frac{\dot{a}}{a}$, they are then written:

$$\left|\begin{array}{l}(1.1)\;H^2=\frac{8\pi G}{3}\rho+\frac{c^2\Lambda}{3}\\ (1.2)\;H^2+\dot{H}=-\frac{4\pi G}{3}\left(1+3\omega\right)\rho+\frac{c^2\Lambda}{3}\end{array}\right.$$

and we deduce immediately from (1.1) and (1.2) the complementary equation, [3]:

$$(1.3):\ \frac{\dot{\rho}}{\rho}=-3\left(1+\omega\right)H$$

1.1 Solving the Friedmann-Lemaître equations:

We assume that $\omega\neq-1$ and we initialize classically the unknowns at time 0, origin of cosmic time corresponding to the date of the last determination of the Hubble parameter. (1.3) and (1.2) give us immediately:

$$\rho\left(\tau\right)=\rho_0\,e^{-3(1+\omega)\int_0^\tau H(s)ds}\quad,\quad\rho_0=\frac{3}{8\pi G}\left(H_0^2-\frac{\Lambda c^2}{3}\right)\qquad(1.4)$$

Insertion of (1.1) into (1.2) gives then $\dot{H}=-4\pi G\left(1+\omega\right)\rho$ and next $\ddot{H}=-4\pi G\left(1+\omega\right)\dot{\rho}$
Hence, given (1.3), we obtain : $\ddot{H}/\dot{H}=\dot{\rho}/\rho=-3\left(1+\omega\right)H$,i.e. $\ddot{H}=-3/2\left(1+\omega\right)(\dot{H^2})$ (1.5)
It follows then from (1.4) and (1.2) that:

$$\dot{H}=\frac{-3\left(1+\omega\right)}{2}H^2+\frac{3\left(1+\omega\right)}{2}\frac{\Lambda c^2}{3}\quad(1.6)$$

Let $H=h+\beta,\ \beta=\sqrt{\Lambda c^2/3}$ (1.9), we obtain: $-\dot{h}/h^2=3\left(1+\omega\right)\beta\,1/h+3/2\left(1+\omega\right)$, i.e., with $\chi=1/h$, $\dot{\chi}=\ 3\left(1+\omega\right)\beta\,\chi+3/2\left(1+\omega\right)$ and then: $\chi=\left(1/2\beta+\chi\left(0\right)\right)e^{3(1+\omega)\beta\,.\,\tau}-1/2\beta$
From that, we deduce immediately, given $H\left(0\right)=H_0$:

$$H\left(\tau\right)=\beta\frac{\left(H_0+\beta\right)e^{3(1+\omega)\beta\,.\,\tau}+\left(H_0-\beta\right)}{\left(H_0+\beta\right)e^{3(1+\omega)\beta\,.\,\tau}-\left(H_0-\beta\right)}\quad(1.7)$$

Note : This formula remains correct for $\omega=-1$: $\ H_{\omega=-1}\left(\tau\right)=H_0,\ \forall\tau$
Furthermore, we obtain then immediately:

$$\dot{H}=\frac{-6\left(1+\omega\right)\beta^2\left(H_0^2-\beta^2\right)e^{3(1+\omega)\beta\,\tau}}{\left(\left(H_0+\beta\right)e^{3(1+\omega)\beta\,\tau}-\left(H_0-\beta\right)\right)^2}\,(1.9)\,,\quad\rho=\frac{\frac{3}{2\pi G}\beta^2\left(H_0^2-\beta^2\right)e^{3(1+\omega)\beta\,\tau}}{\left(\left(H_0+\beta\right)e^{3(1+\omega)\beta\,\tau}-\left(H_0-\beta\right)\right)^2}\quad(1.10)$$

1.2 Condition of positivity and singularities::

As $H > \beta, cf(1.1)$, $H(\tau) > 0$ is equivalent to:

$$(1+\omega)\ \tau > (1+\omega)\,\tau_\omega\ with\ \tau_\omega = \frac{1}{3\,(1+\omega)\,\beta}\ln\left(\frac{H_0-\beta}{H_0+\beta}\right)\ (1.8)$$

Basic cases:

$\omega > -1$:

$\omega = 0$: Universe dominated by dust gas / $\omega = {}^1\!/_3$: Universe dominated by radiations H is defined and decreasing for all $\tau > \tau_\omega$. τ_ω is, a priori, the cosmic date of the Big Bang relatively to the chosen time origin.

$$\lim_{\tau_\omega} H = +\infty\ ,\ \lim_{\tau_\omega} \overset{\bullet}{H} = -\infty \quad \& \quad \lim_{+\infty} H\ = \beta\ = \sqrt{\frac{\Lambda c^2}{3}},\ \ \lim_{+\infty} \overset{\bullet}{H} = 0$$

The Universe is in perpetual extension.

$\omega = -1$: Universe dominated by emptyness.

$H(\tau) = H_0,\ \forall\tau \in \mathbb{R}$: the Universe is uncreated, with no singularity and then not admits a Big Bang following the first postulate.

$\omega < -1$: Universe dominated by *exotic matter (R.Caldwell - A Phantom Menace? Cosmological consequences of a dark energy component with super-negative equation of state; Physics Letters B, no 545,? 2002)*

H is defined for all $\tau < \tau_\omega$:its only singularity is in its future and not in its past and, hence, Universe without Big Bang.

More generally, if we consider an Universe of constituents $i = 1, 2; ... / p_i = \omega_i c^2 \rho_i$, the Friedmann-Lemaître equations are unchanged setting $\rho = \sum_i \rho_i\ et\ \omega = \sum_i \left({}^{\rho_i}\!/_{\rho}\right)\omega_i$

Furthermore, it is proved (cf [2],[1]) that the only case corresponding to all our required conditions is $\omega > -{}^1\!/_3$ and not $\omega > -1$. We will find again this restriction in section 2.5. and admit it for the moment.

2.Draft of a periodic Cosmos

In the continuation of this essay, we will always suppose that $\omega > -1/3$

Furthermore, in order to avoid any ambiguity, we will reserve the notation $\overset{\bullet}{w}$ to the derivation relating to cosmic time and will denote w' that relating to apparent time.

2.1 The Cosmic time Curve As announced in the introduction, we assume here that the discrepancy appeared in the evolution of the Hubble parameter is due to an impossibility to identify cosmic and apparent times for all time and, to solve this problem, that bring us to build a model of periodic Cosmos as it follows:

2^{nd} ***Postulate***:*the most fundamental curve of Astrophysics, the ellipsis, necessary emanates from the most fundamental curve of the Cosmos, its Curve of (cosmic) time $\mathcal{T}$.*

Given the irreversibility of time, we deduce then that the infinite Curve of time $\mathcal{T}$ is the periodic elliptic curve defined below:

Let $(O, \overrightarrow{i}, \overrightarrow{j})$ an orthonormal system of $\mathbb{R}^2$: coordinates $(x = ct, y)$ and $1 << T$.

We denote $\mathcal{T}_0$ the half ellipsis defined by: $\left({}^{t}\!/_{T}\right)^2 + y^2 = 1\ ,\ y \geqslant 0$ and, subject to consistence, we will consider that:

$\mathcal{T}_0$ is the curve of time corresponding to our Universe.

The axis $\left(O,\ \overrightarrow{i}\right)$ is the apparent time axis (modulo c)

then:

$t = -T\ \ (x = -cT)$ is the apparent "birth date" of our Universe (Big Bang) and $t = T\ \ (x = cT)$ is the apparent date of its "death date" (Big Chaos / see 2.5)

Let $\mathcal{P} : t \in [-T, T] \to (ct, y(t)$, with $y\,(t) = \sqrt{1 - \left(\frac{t}{T}\right)^2}$ (2.1) the natural setting of $\mathcal{T}_0$.
We define then the curve of cosmic time $\mathcal{T}$ in its totality as the periodic curve of setting $\mathcal{P} : t \in \mathbb{R} \to (ct, y(t))$:

$$y\,(t) = y\,(t - 2kT)\ \ if\ \ t \in [(2k-1)\,T, 2\,(k+1)\,T] = [t_k,\ t_{k+1}] \quad (2.2)$$

$\mathcal{P}$ is a homeomorphism from $\mathbb{R}$ on $\mathcal{T}$ which points of apparent time $t_k = (2k-1)\,T,\ k \in \mathbb{Z}$ are cusps.

If t is an apparent time and $\tau(t)$ the cosmic time, proper time of $\mathcal{T}$, which corresponds to it, we have:

τ and$t$ exactly coincide only at their common origin O.

$\tau(t) = \ \ \tau_t = \int_0^t \gamma\,(s)\ \ ds$ with $\ \gamma\,(t) = \frac{d\tau}{dt} = \sqrt{1 + y'\,(t)^2} \quad (2.3)$

Furthermore :

$\forall t :\ \ \tau\,(-t) = -\tau\,(t)$

if $t \in [0, T],\ \ \tau\,(t) = T.\int_0^{Arc\sin\left({}^{t}\!/_{T}\right)} \sqrt{1 - e^2 \sin^2 \alpha}\,.\,d\alpha \quad with\ \ e = \sqrt{1 - \left({}^{1}\!/_{T_\omega}\right)^2}$ (2.4)

$\tau() : t \to \tau_t$ is a homeomorphism from $\mathbb{R}$ onto itself, and a smooth diffeomorphism from $\mathbb{R} - \{t_k,\ k \in \mathbb{Z}\}$ onto $\mathbb{R} - \left\{\tau^k = \tau\,(t_k)\,,\ k \in \mathbb{Z}\right\}$

The *Cosmos*, or global Universe, is then defined as the infinite succession of (local) Universes $U_k,\ k \in \mathbb{Z}$, each equipped with the FLRW metric of zero spatial curvature for the coordinates system $(\tau, r, \theta, \phi)\,,\ \ \tau \in \left]\tau_k, \tau_{k+1}\right[$:

$$g_{\alpha,\beta} dx^\alpha dx^\beta = -\,(cd\tau)^2 + a\,(\tau)^2 \left[dr^2 + r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

completed by the state equation $p = \omega c^2.\rho,\ \ \omega > {}^{-1}\!/_{3}$

The Cosmos would be then uncreated, infinite succession of (local) Universes spatially homogeneous, isotropic and identical for great scales, of same *half-life* T in apparent time and $\int_0^T \gamma\,(s)\,.\,ds \quad = T\int_0^{\pi/2} \sqrt{1 - e^2 \sin^2 \alpha}\ d\alpha$ (2.5) in cosmic time; we will denote it $\mathcal{T}.\Sigma_\tau$

We have now to justify interest and consistency of a such cosmological model:

2.2 Hubble parameter in cosmic time

Obviously, the Hubble parameter is here initialized relatively to the (common) absolute Origin O of cosmic time (and apparent time) and its initial value will be denoted H_O. So, to avoid any ambiguity, all values relating to the dating used in section 1. will from now on be noted: $H_0^*, \tau_\omega^*, ...$

We have to note that, for the moment, all parameters, T, O, H_O, ... of our model are purely theoretical, their values relative to usual measures will can be only determined in the end of this essay (2.5 and Annexe).

For our Universe U_0, the Hubble parameter is then given by (1.7):

$$\forall\tau\in\left]\tau_0,\tau_1\right[,\quad H(\tau)=\beta\frac{(H_O+\beta)\,e^{3(1+\omega)\beta\,.\,\tau}+(H_O-\beta)}{(H_O+\beta)\,e^{3(1+\omega)\beta\,.\,\tau}-(H_O-\beta)}\quad(2.6)$$

and the theoretical date of the Big Bang is then given by (1.8):

$$(2.7)\qquad \tau_\omega=\frac{1}{3(1+\omega)\beta}\ln\left(\frac{H_O-\beta}{H_O+\beta}\right)$$

It follows immediately that the half-life of the successive Universes is $|\tau_\omega|$ in cosmic time and T_ω in apparent time where T_ω is the only solution to the elliptic equation

$$|\tau_\omega|=T.\int_0^{\pi/2}\sqrt{1-\left(1-{}^1\!/\!_{T^2}\right)\sin^2\alpha}\,.\,d\alpha\quad(2.8)$$

The Hubble parameter is then defined for all cosmic time τ, out of singularities, by $2|\tau_\omega|$-periodicity:

$$H(\tau)=H(\tau')\ ,\ \tau'\in\left]\tau_\omega,|\tau_\omega|\right[\ /\ \tau=\tau' modulo\,(2|\tau_\omega|)\quad(2.9)$$

and in each singularity: $\lim\limits_{(\tau^k)^-}H=\lim\limits_{|\tau_\omega|^-}H=\frac{(H_O^2+\beta^2)}{2H_O}\quad,\quad\lim\limits_{(\tau^k)^+}H=\lim\limits_{\tau_\omega^+}H=+\infty\quad(2.10)$

Following the same process and using (1.10) the density, out of singularities, is given by:

$$\rho(\tau)=\frac{3}{2\pi G}\beta^2\frac{(H_O^2-\beta^2)\,e^{3(1+\omega)\beta\,\tau'}}{\left((H_O+\beta)\,e^{3(1+\omega)\beta\,\tau'}-(H_O-\beta)\right)^2},\quad\tau'\in\left]\tau_\omega,|\tau_\omega|\right[\ /\ \tau\equiv\tau'\bmod(2|\tau_\omega|)\quad(2.11)$$

and in each singularity: $\lim\limits_{(\tau^k)^-}\rho=\frac{3}{4\pi G}\beta\frac{(H_O+\beta)^2(H_O-\beta)}{H_O}\quad,\quad\lim\limits_{(\tau^k)^+}\rho=+\infty\quad(2.12)$

2.3 Hubble parameter in apparent time

We define the Hubble parameter in apparent time by $\overline{H}(t)={}^{\overline{a}'}\!/\!_{\overline{a}}(t)$, with $\overline{a}(t)=a(\tau_t)$ (2.13) and we have then:

$$\overline{H}'(t)=\gamma'(t)\,H(\tau_t)+\gamma^2(t)\,\overset{\bullet}{H}(\tau_t)\quad(2.14)$$

Consequences:

(i) *for* $|t|<<T_\omega$: $\overline{H}(t)\approx H(\tau_t)\quad,\quad\overline{H}'(t)\approx\overset{\bullet}{H}(\tau_t)\quad(2.16)$

(ii) $\lim\limits_{T_\omega^-}\overline{H}=\lim\limits_{T_\omega^-}\gamma\,.\,\lim\limits_{|\tau_\omega|^-}H=+\infty\quad,\quad\lim\limits_{(-T_\omega)^+}\overline{H}=\lim\limits_{(-T_\omega)^+}\gamma\,.\,\lim\limits_{\tau_\omega^+}H=+\infty\quad(2.17)$

$$(iii)\quad\overline{H}'(t)\underset{T_\omega^-}{\sim}\left({}^1\!/\!_{T_\omega}\right)^2\frac{1}{\sqrt{1-\left({}^t\!/\!_{T_\omega}\right)^2}^{\,2}}\left(\frac{1}{\sqrt{1-\left({}^t\!/\!_{T_\omega}\right)^2}}.\underbrace{H\left(|\tau_\omega|^-\right)}_{>0}+\underbrace{\overset{\bullet}{H}\left(|\tau_\omega|^-\right)}_{\in\mathbb{R}}\right)$$

then : $\lim\limits_{T_\omega^-}\overline{H}'=+\infty\quad(2.19)$

$$(iv)\quad \overline{H}'(t) \underset{(-T_\omega)^+}{\sim} \frac{-\left(1/T_\omega\right)^2}{\sqrt{1-\left(t/T_\omega\right)^2}^{2}} \frac{1}{(H_O+\beta)\,e^{3(1+\omega)\beta\,.\,\tau}-(H_O-\beta)} \times$$

$$\left(\frac{\overbrace{\lambda}^{>0}}{\sqrt{1-\left(t/T_\omega\right)^2}} + \frac{\overbrace{\mu}^{>0}}{(H_O+\beta)\,e^{3(1+\omega)\beta\,.\,\tau}-(H_O-\beta)} \right)$$

$$then: \qquad \lim_{(-T_\omega)^+} \overline{H}' = -\infty \quad (2.20)$$

As $\overline{H}'(O) = \overset{\bullet}{H}(O) < 0$, it follows that, on $]t_0, t_1[\,=\,]-T_\omega, T_\omega[$, the Hubble parameter in apparent time is first decreasing until a time $t_{\min} > O$ (2.19) and then increasing: the discrepancy is solved.
Furthermore, we have $H_0^* = \overline{H}(t_H)$ with $t_{H_0^*} > t_{\min}$ (2.20)

2.4. Consistency with the natural metric in apparent time of the (global) space-time

The natural metric in apparent time of the Cosmos is clearly the ”local” modified FLRW metric:
$\forall U_k:\ \overline{g}^k_{\alpha,\beta}dx^\alpha dx^\beta = -\gamma^2(t)\,d(ct)^2 + \overline{a}^2\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]\ t \in\,]t_k, t_{k+1}[$ (2.21)
completed by the state equation $\overline{p} = \omega c^2 \overline{\rho}$, $\omega > {-1}/{3}$ with $\overline{\rho}(t) = \rho(\tau_t)$ and $\overline{p}(t) = p(\tau_t)$, (2.22)

Not null Christoffel symbols:

$$\overline{\Gamma}^0_{00} = \left(1/c\right)\frac{\gamma'}{\gamma}\ ,\ \overline{\Gamma}^0_{rr} = \left(1/c\right)\frac{\overline{a}\,\overset{\bullet}{\overline{a}}}{\gamma^2},\ \overline{\Gamma}^0_{\theta\theta} = \left(1/c\right)\frac{\overline{a}\,\overset{\bullet}{\overline{a}}}{\gamma^2}r^2\ ,\ \overline{\Gamma}^0_{\phi\phi} = \left(1/c\right)\frac{\overline{a}\,\overset{\bullet}{\overline{a}}}{\gamma^2}r^2\sin^2\theta$$

$$\overline{\Gamma}^r_{0r} = \overline{\Gamma}^r_{r0} = \left(1/c\right)\frac{\overset{\bullet}{\overline{a}}}{\overline{a}}\quad ,\quad \overline{\Gamma}^r_{\theta\theta} = -r\quad ,\quad \overline{\Gamma}^r_{\phi\phi} = -r\sin^2\theta$$

$$\overline{\Gamma}^\theta_{0\theta} = \overline{\Gamma}^\theta_{\theta 0} = \left(1/c\right)\frac{\overset{\bullet}{\overline{a}}}{\overline{a}}\quad ,\quad \overline{\Gamma}^r_{r\theta} = \overline{\Gamma}^r_{\theta r} = \frac{1}{r}\quad ,\quad \overline{\Gamma}^\theta_{\phi\phi} = -\sin\theta\cos\theta$$

$$\overline{\Gamma}^\phi_{0\phi} = \overline{\Gamma}^\phi_{\phi 0} = \left(1/c\right)\frac{\overset{\bullet}{\overline{a}}}{\overline{a}}\quad ,\ \overline{\Gamma}^r_{r\phi} = \overline{\Gamma}^r_{\phi r} = \frac{1}{r}\quad ,\ \overline{\Gamma}^\phi_{\theta\phi} = \overline{\Gamma}^\phi_{\phi\theta} = \frac{1}{\tan\theta}$$

Not null Ricci tensor components:

$$\overline{R}_{00} = \left(3/c^2\right)\left(\frac{\gamma'}{\gamma}\frac{\overline{a}'}{\overline{a}} - \frac{\overline{a}''}{\overline{a}}\right)$$

$$\overline{R}_{rr} = \left(1/c^2\right)\frac{1}{\gamma^2}\left(\overline{a}''\overline{a} + 2\overline{a}'^2 - \frac{\gamma'}{\gamma}\overline{a}\,\overline{a}'\right)$$

$$\overline{R}_{\theta\theta} = r^2.\overline{R}_{rr}\quad ,\qquad \overline{R}_{\phi\phi} = r^2\sin^{2\theta}.\,\overline{R}_{rr}$$

Curvature scalar:

$$\overline{R} = \left(6/c^2\right)\frac{1}{\gamma^2}\left(\frac{\overline{a}''}{\overline{a}} + \left(\frac{\overline{a}'}{\overline{a}}\right)^2 - \frac{\gamma'}{\gamma}\frac{\overline{a}'}{\overline{a}}\right)$$

Not null energy-impulse tensor components:

$$\overline{T}_{00} = \overline{\rho}\, c^2\gamma^2 \ , \quad \overline{T}_{rr} = \omega c^2 \overline{\rho}\, \overline{a}^2 \ , \quad \overline{T}_{\theta\theta} = \overline{T}_{rr} r^2, \quad \overline{T}_{\phi\phi} = \overline{T}_{rr} r^2 \sin^2\theta$$

The Einstein equations system in apparent time is then reduced to the two following independents equations:

$$\left| \begin{array}{ll} 00: & \left(\dfrac{\overline{a}'}{\overline{a}}\right)^2 = \gamma^2 \left(\dfrac{8\pi G}{3}\overline{\rho} + \dfrac{\Lambda c^2}{3}\right) \\ rr: & \dfrac{\overline{a}''}{\overline{a}} + \dfrac{1}{2}\left(\dfrac{\overline{a}'}{\overline{a}}\right)^2 - \dfrac{\gamma'}{\gamma}\dfrac{\overline{a}'}{\overline{a}} = \gamma^2 \left(\dfrac{-4\pi G}{c^2}\omega\overline{\rho} \ + \dfrac{\Lambda c^2}{2}\right) \end{array} \right.$$

i.e. by inserting 00 in rr and writing the system with $\overline{H} = {\overline{a}'}/{\overline{a}}$:

$$\left| \begin{array}{l} (2.24) \quad \overline{H}^2 = \gamma^2 \left(\dfrac{8\pi G}{3}\overline{\rho} + \dfrac{c^2\Lambda}{3}\right) \\ (2.25) \quad \overline{H}' + \overline{H}^2 - \dfrac{\gamma'}{\gamma}\overline{H} = \gamma^2 \left(\dfrac{-4\pi G}{c^2}\omega\overline{\rho} \ + \dfrac{\Lambda c^2}{2}\right) \end{array} \right. \quad , \qquad t \in]t_k, t_{k+1}[$$

Let us now replace $\overline{H}$, $\overline{H}'$ and by their expressions (2.12), (2.13) and (2.22) respectively, we find again the initial Friedmann-Lemaître system in $(H(\tau_t), \rho(\tau_t))$ for all $t \in]t_k, t_{k+1}[$, i.e. for all $\tau \in]\tau^k, \tau k + 1[$ which (H, ρ) is the only solution. Hence the cosmological model $\mathcal{T}.\Sigma$ is consistent with the natural metric of the global space-time.

2.5 The Hubble Law in the vicinity of the singularity $\tau^k \ / \ t_k$

Because of periodicity, it is sufficient to do our study on the local Universe U_0.
We will do here a dynamic interpretation of the Hubble Law relatively to the second Newton Law on vicinities ${\tau_\omega}^+$ and $|\tau_\omega|^-$.
For that, we must take care to the two followings points:

(1) The Hubble Law is fundamentally a law in cosmic time since the Hubble parameter itself directly emanates from the Friedmann-Lemaître equations. The Hubble parameter in apparent time only expresses the greater or lesser concentration of cosmic time units per apparent time unit following the proper curvature of the cosmic time and, moreover, the scale factor functions a and $\overline{a}$ are fundamentally the same, just taken on different times: $\overline{a}(t) = a(\tau)$ with $\tau = \tau_t \neq t (t \neq 0)$.

The Hubble law is an expansion law to the future and hence its dynamic interpretation must respect the arrow time: If in the vicinity $|\tau_\omega|^-$, time goes to the future and the arrow time is then respected; at the opposite, in its vicinity ${\tau_\omega}^+$, time goes to the past and we must to redirect correctly the arrow time to the future.
More generally, we will consider that positive cosmic times are in the "future"of the Universe U_0 and the negative are in its "past", the border between past and future being then the absolute origin 0.

Let us then consider a two objects system $A + B$ of respective masses m_A, m_B , and of infinitesimal centres of mass distance $AB = \delta l$. For all couple in its validity domain, the expansion Hubble Law can be interpreted as expressing the recession velocity of B relative to A at time :

$$\overrightarrow{v}_{B/A}(\tau) = H(\tau) . \overrightarrow{\delta l} ., \ \ with \ \ \overrightarrow{\delta l} = \delta l . \frac{\overrightarrow{AB}}{AB} \quad (2.23)$$

Following the second Newton Law, A induces on B a repulsive force that we will name *Hubble force induced by A on B* and expressed by (2.24):

$$\overrightarrow{f}_{B/A}(\tau) = \varepsilon(\tau)\,.\,\frac{d\left(m_B \overrightarrow{v}_{B/A}\right)}{d\tau}(\tau) = \varepsilon(\tau)\, m_B \left(\overset{\bullet}{H}(\tau) + H^2(\tau)\right).\, \overrightarrow{\delta l} \ \ ,\ \varepsilon = \left| \begin{array}{l} +1 \ for\ \tau > O \\ -1 \ for\ \tau < O \end{array} \right.$$

Study of the condition $\varepsilon(\tau)\left(\overset{\bullet}{H}(\tau) + H^2(\tau)\right) > 0$:

Following (2.6) and (1.9), we obtain:

$$\varepsilon(\tau)\left(\overset{\bullet}{H}(\tau) + H^2(\tau)\right) \underset{\tau_\omega^+}{\approx} (-1)\ \beta^2 \frac{-2(1+3\omega)(H_0-\beta)^2}{\left((H_0+\beta)\, e^{3(1+\omega)\beta\ \tau_t} - (H_0-\beta)\right)^2} > 0 \ \ (2.25)$$

if, and only if $\omega > {}^{-1}\!/_{3}$

As previous, we have found again the condition admitted in 1.2

Furthermore, it is clear, following (2.25) and 1.2 / $\omega > -1$ respectively, that

$$\lim_{\tau_\omega^+}\left(\overset{\bullet}{H} + H^2\right) = -\infty \quad and \quad \lim_{|\tau_\omega|^-}\left(\overset{\bullet}{H} + H^2\right) \approx \beta^2 > 0$$

On the other hand, as $\overset{\bullet}{\left(\overset{\bullet}{H} + H^2\right)} = \overset{\bullet\bullet}{H} + 2H\overset{\bullet}{H} = -(1+3\omega)\, H\, \overset{\bullet}{H} > 0$, $\overset{\bullet}{H} + H^2$ is always increasing. We deduce from this that it exists one and only one cosmic time for which $\overset{\bullet}{H} + H^2 = 0$, (2.26) and that time is O. For that, we define then the value of H_O as the only positive solution of the equation (2.26), i.e., following (1.7),(1.8) $-6(1+\omega)\beta^2\left(H_O^2 - \beta^2\right) + 4H_O^2 = 0$ and hence:

$$H_O = \sqrt{\frac{3(1+\omega)}{1+3\omega}}\beta \quad (2.27)$$

It follows that all parameter values of the Cosmos are totally determined by the Cosmological Constant Λ and the global state parameter ω.

2.5.1 The singularity $|\tau_\omega|^-$ / T_ω^-: the Big Chaos

Let us consider again the preceding two points system $A + B$ in the vicinity $|\tau_\omega|^-$: it follows from (2.25) that the instant power of the Hubble force is given by:

$$\Delta E(\tau) = m_B.\, \varepsilon(\tau)\left(\overset{\bullet}{H}(\tau) + H^2(\tau)\right)\ \delta l^2\, d\tau \underset{\left(\tau^{(k)}\right)^-}{\approx} \lim_{|\tau_\omega|^-}\left(\overset{\bullet}{H} + H^2\right) .\delta l^2\, d\tau \quad (2.27)$$

and then, in apparent time:

$$\Delta\overline{E}(t) = \Delta E(\tau_t)\,.\frac{d\tau}{dt}.dt \underset{T_\omega^-}{\approx} \lim_{|\tau_\omega|^-}\left(\overset{\bullet}{H} + H^2\right).\delta l^2.\gamma(t)\,.dt \underset{t \overset{<}{\to} T_\omega}{\to} +\infty \quad (2.28)$$

The instant energy transmitted in apparent time to the system $A + B$ by the Hubble force will therefore reach the fission activation threshold when $t \overset{<}{\to} T_\omega$ for any binding energy ensuring the cohesion of system $A + B$, whether of gravitational or nuclear origin, ([4]), and that suggests the following process:

In a first time, breaking of cohesions by gravitational forces :crumbling of stellar formations: planets... stars... galaxies...black holes

next, for t near of T_ω, breaking of cohesions by nuclear forces: fission of atoms...atomic particles.. subatomic particules... particles / antiparticules

The universe ends in a thermonuclear chaos which ultimate phase is the fission particles/ antiparticles.

2.5.2 The singularity τ_ω^+ / $(-T_\omega)^+$: the Big Bang [4],[1]

At the opposite of the above situation where effects of the Hubble Law would be "negligible" ($H \approx \sqrt{\Lambda c^2/3}$) without cosmic time curvature, it is not the same here: in the Universe U_{-1}, when $\tau \xrightarrow{<} \tau_\omega$, the energy produced by each fission preceding the ultimate participated both to the temperature increase of the universe and the start of the following fission phase. The quasi-infinite energetic flash due to the ultimate fission induces, in each point, at cosmic time τ_ω a colossal contribution ΔE of energy both thermic ΔTh and kinetic ΔE_k. This last induces a recession velocity $\overrightarrow{v}_{B/A}$ of each ultimate constituent B relatively to each other A such as $\overrightarrow{AB} = \overrightarrow{\delta l}$ be infinitesimal:

$$\Delta E_k = m_B.v_{B/A}^2 = m_B.(H\delta l)^2 \quad (2.29)$$

which induces then an "infinite" value of the Hubble parameter in cosmic time, but nevertheless negligible compared to its value in apparent time.
The ultimate thermonuclear cataclysm "creates" an "infinite" value of the cosmic time Hubble parameter and, so, induces the inflation phenomena, the curvature of the cosmic time only amplifies it but does not "create" it.
The contribution of fusion energy due to the nucleosynthesis is very brief and next , without new energetic contribution, the H value falls rapidly as previous by its theoretical expression.
More precisely that suggests the following diagram:

The Big Chaos in ${\tau_\omega}^-$ induces the following consequences in ${\tau_\omega}^+$:

(a) "infinite" H value ($\leftarrow$ "infinite" energetic flash / (2.29)) $\rightarrow \overline{H} >> H$ (2.30)
(b) particles/antiparticles gas $\rightarrow$ plasma ($\leftarrow$ "infinite" electromagnetic flash) with ultra high temperature ($\leftarrow$ "infinite" thermic flash) and density ($\leftarrow$ "infinite" H value / (1.4)) $\rightarrow$ Nucleosynthesis

and then: Nucleosynthesis + ultra high density + (2.30) $\rightarrow$ "infinite" expansion velocity in apparent time: inflation
The Big Chaos in $(-\tau_\omega)^-$ creates en $(\tau_\omega)^+$ the initial conditions of the Big bang.

So, the Cosmos flares up and is reborn "identical to itself" like a phoenix at each singularity of its curve of time.

Annexe: Relations between observational data and absolute dating

Observational data are denoted (apparent time):

$$\widetilde{H_0^*} = \gamma\left(t_0\right) H\left(\tau_0 = \tau\left(t_0\right)\right) = \gamma\left(t_0\right) H_0^*, \quad \widetilde{\rho_0^*} = \rho_0^* = \rho\left(\tau_0\right) = \widetilde{\rho}\left(t_0\right), \quad \widetilde{\Omega_\Lambda^*} = \beta^2 \Big/ \widetilde{H_0^*}^2, \ \widetilde{\Omega^*} = \frac{8\pi G}{3\,\widetilde{H_0^*}^2}\rho_0^*, \ ...$$

Assuming the global state parameter ω and the cosmological Constent determined from the above datas,we obtain then successively:

$H_O = \sqrt{\frac{3(1+\omega)}{1+3\omega}\beta}$, $\tau \to H(\tau)$ and $\tau \to \rho(\tau)$,

$\tau_\omega = \frac{1}{3(1+\omega)\beta} \ln\left(\frac{H_O-\beta}{H_O+\beta}\right)$, T_ω, such as $T_\omega \int_0^{\pi/2} \sqrt{1 - \left({}^1\!/_{T_\omega}\right)^2 \sin^2\,\alpha}\,.\,d\alpha = |\tau_\omega|$

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